

x1: Algebraic Topology

- (1) Let X be a 5-manifold with fundamental group $H = \langle a; b^2; bab^{-1} \rangle$. Is H a normal subgroup or not? How do you see this from the covering space?
- (2) Prove that if M is a compact 3-dimensional submanifold of S^3 , then $H_1(M; \mathbb{Z})$ is torsion-free.
- (3) Prove that a continuous mapping from the 17-dimensional unit ball to itself fixes some point.
- (4) (a) Describe a cell decomposition of $\mathbb{R}P^n$ involving one cell of each dimension from 0 to n inclusive.
 (b) Write down the associated cell chain complex of $\mathbb{R}P^5$ with \mathbb{Z}