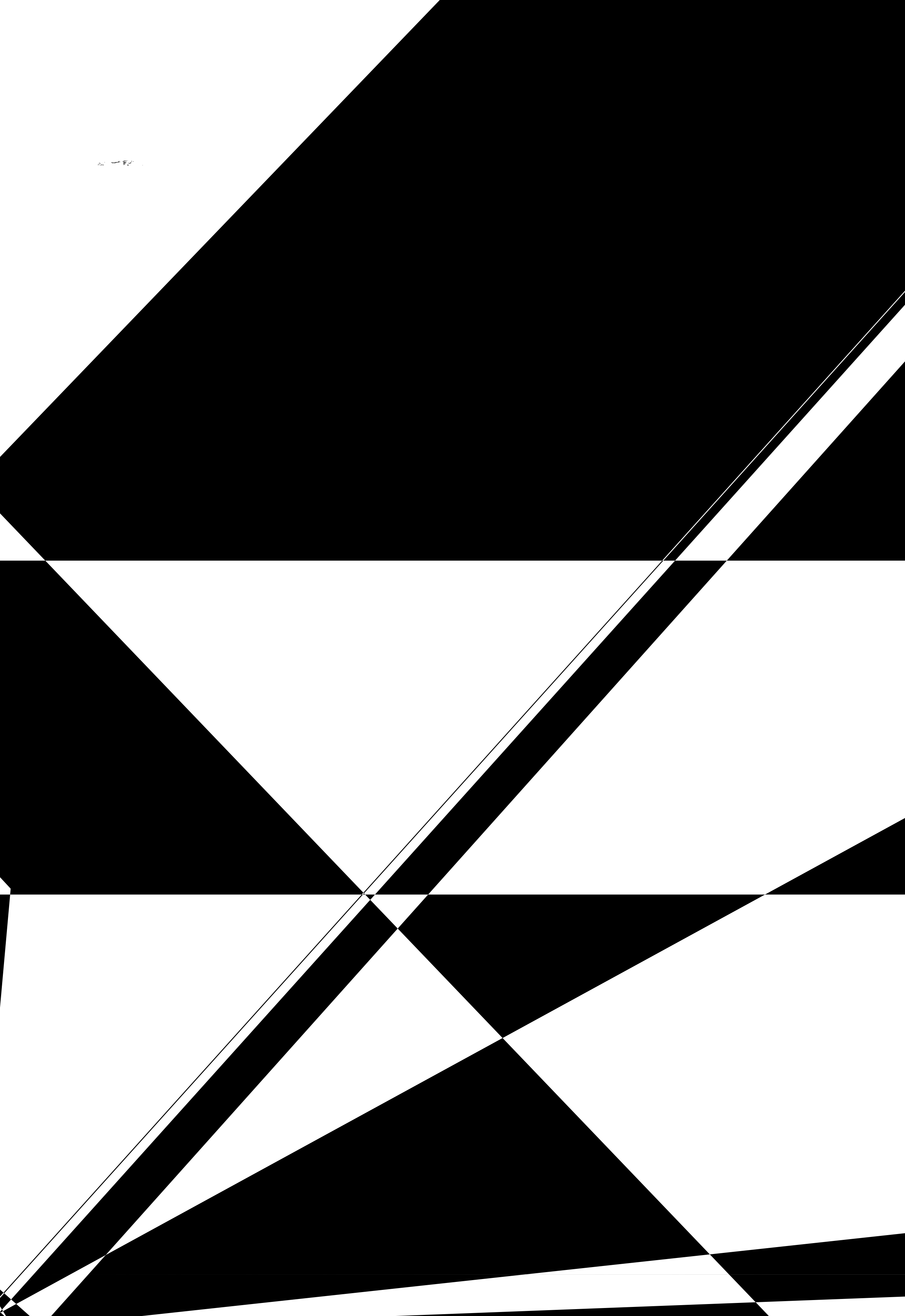

[illegible] $X = (x_1, s_1; \ldots; x_n$

. (3) .

x X Y, Y. Y Y' x

Lemma 1. *Let $\succeq$ be a distribution-regret preference relation. Then $\succeq$ admits a two-dimensional regret function* $^*: \mathcal{D} \times \mathcal{D} \to \Re$ *and a regret functional V^* such that*

$$
\begin{aligned}
X \succeq Y \quad &\Longleftrightarrow \quad V^*\big(\;^*(x_1, c_Y), p_1; \ldots; \;^*(x_n, c_Y), p_n\big) \geqslant 0 \\
&\Longleftrightarrow \quad V^*\big(\;^*(y_1, c_X), q_1; \ldots; \;^*(y_m, c_X), q_m\big) \leqslant 0
\end{aligned}
$$

where c_X and c_Y are the certainty equivalents of X and Y respectively.

Proof. V $\succeq$. $y \in \mathcal{D}$, $^*(x, y) = (x, \delta_y)$, δ_y, y 1,

$$
V^*\big(\;^*(x_1, y), p_1; \ldots; \;^*(x_n, y), p_n\big) = V\big((x_1, \delta_y), p_1; \ldots; (x_n, \delta_y), p_n\big)
$$

, ,

$$
\begin{aligned}
X \succeq Y \quad &\Longleftrightarrow \quad X \succeq \delta_{c_Y} \\
&\Longleftrightarrow \quad V\big((x_1, \delta_{c_Y}), p_1; \ldots; (x_n, \delta_{c_Y}), p_n\big) \geqslant 0 \\
&\Longleftrightarrow \quad V^*\big(\;^*(x_1, c_Y), p_1; \ldots; \;^*(x_n, c_Y), p_n\big) \geqslant 0 \qquad \square
\end{aligned}
$$

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1,

Definition 4. $\succeq$ *distribution-regret based*

$: \mathcal{D} \times \mathcal{D} \to \Re$,

$V : \to \Re$

$$X \succeq Y \quad \textit{iff} \quad V\big(\Psi(X, c_Y)\big) \geqslant 0 \quad \textit{iff} \quad 0 \geqslant V\big(\Psi(Y, c_X)\big),$$

$\Psi(X, c_Y) = (\ (x_1, c_Y), p_1; \ldots; \ (x_n, c_Y), p_n)$ X δ_{c_Y} (Y), $\Psi(Y, c_X)$ Y δ_{c_X} (X),

x

$$X \sim \delta_{c_Y} \implies V\Big(\big(n\, d \qquad V$$

[illegible]

Proposition 3. *If the preference relation $\succeq$ is consistent then it satisfies distribution regret.*

Proof. [illegible] $Z \in \mathcal{L}$ [illegible] $Z = \delta_x$ [illegible] x [illegible] $\mathcal{D}$), [illegible] $\lambda(Z)$ [illegible] λ [illegible] $f(c_Z, \lambda) = 0$.[10] [illegible] 5, [illegible] $\lambda(Z)$ [illegible] $Z \in \mathcal{L}$, [illegible] $Z = \delta_x$ [illegible] x [illegible] $\mathcal{D}$. [illegible] U [illegible] $\succeq$ [illegible] $U(\delta_x) = x$ [illegible] x. [illegible] $(x, c_Y) = f(x, \lambda(Y))$.[11] [illegible] (x, c_Y) [illegible] x [illegible] f [illegible] $\lambda(Y)$ [illegible] $\lambda(Y)$ [illegible] Y [illegible] 0. [illegible] $\succeq$ [illegible] $\lambda(Z) = -c_Z$ [illegible] $(x, y) = x - y$. [illegible] 5, [illegible] $x \in \mathcal{D}$ [illegible] $f(x, \lambda(Y)) \in \mathcal{D}$. [illegible]

$$f\big(X, \lambda(Y)\big) = \big(f\big(x_1, \lambda(Y)\big), p_1; \ldots; f\big(x_n, \lambda(Y)\big), p_n\big)$$
$$= \big(\ (x_1, c_Y), p_1; \ldots; \ (x_n, c_Y), p_n\big)$$
$$= \Psi(X, c_Y) \tag{4}$$

[illegible]

$$X \succeq Y \sim \delta_{c_Y} \iff f\big(X, \lambda(Y)\big) \succeq f\big(Y, \lambda(Y)\big) \sim \delta_{f(c_Y, \lambda(Y))} = \delta_0$$
$$\iff U\big(f\big(X, \lambda(Y)\big)\big) \geqslant U\big(f\big(Y, \lambda(Y)\big)\big) = U(\delta_0) = 0$$

[illegible]

$$V\big(\Psi(X, c_Y)\big) = U\big(f\big(X, \lambda(Y)\big)\big) = U\big(\Psi(X, c_Y)\big)$$

([illegible] (4)). [illegible]

$$X \succeq Y \iff U\big(\Psi(X, c_Y)\big) \geqslant 0$$
$$\iff V\big(\Psi(X, c_Y)\big) \geqslant 0$$

[illegible] $\succeq$ [illegible]. □

[illegible] ([illegible] 2 [illegible]). [illegible], $u(0) = 0$. $Z = (z_1, r_1; \ldots; z_n, r_n)$ [illegible] $z_1 \leqslant \ldots \leqslant z_n$, [illegible] c_Z [illegible] Z [illegible]

$$c_Z = u^{-1}\left(u(z_1)g(r_1) + \sum_{i=2}^{n} u(z_i)\left[g\left(\sum_{j=1}^{i} p_j\right) - g\left(\sum_{j=1}^{i-1} p_j\right)\right]\right)$$

[illegible] $f(x, \lambda) = u^{-1}(u(x) + \lambda)$ [illegible]

$$f\big(c_Z, \lambda(Z)\big) = 0 \implies \lambda(Z) = -u(z_1)g(r_1) - \sum_{i=2}^{n} u(z_i)\left[g\left(\sum_{j=1}^{i} p_j\right) - g\left(\sum_{j=1}^{i-1} p_j\right)\right]$$

[illegible] U [illegible] $\succeq$ [illegible] $U(\delta_x) = x$ [illegible] $U(Z) = c_Z$ [illegible]. [illegible] (x, c_Y), [illegible] x [illegible] X [illegible] Y, [illegible]

---

10 [illegible] $0 \in$ [illegible] $[\mathcal{D}]$. [illegible] $d \in$ [illegible] $[\mathcal{D}]$, [illegible] $f(c_Z, \lambda(Z)) = d$ [illegible] $(x, x) = d$.

11 [illegible] $(x, x) =$ [illegible] $(x, c_{\delta_x}) = f(x, \lambda(\delta_x)) = 0$.

$$(x, c_Y) = f\big(x, \lambda(Y)\big) = u^{-1}\big(u(x) + \lambda(Y)\big)$$

[illegible] u [illegible] x [illegible] X [illegible] Y [illegible] x [illegible] ([illegible]) [illegible] Y. [illegible] u [illegible].

Example 3. [illegible] $\succeq$ [illegible] $\mathcal{I}$ [illegible].[12] [illegible] [5].

[illegible] $X, Y, Z \in \mathcal{I}$. [illegible] $\succeq$ [illegible] $V(\Psi(X, c_Z)) = V(\Psi(Y, c_Z)) = 0$, [illegible] $\mathcal{I}$, [illegible] $\alpha \in (0, 1)$, $V(\Psi(\alpha X + (1-\alpha)Y, c_Z)) = 0$.

[illegible] Z' [illegible] $\mathcal{I}$ [illegible] X' [illegible] Y' [illegible],

$$\Psi\big(X', c_{Z'}\big) =$$

$$[X]+\alpha\mu_X^+=(1+\alpha)\alpha \quad\Longrightarrow\quad \alpha=\frac{-(1-\mu_X^+)+\sqrt{(1-\mu_X^+)^2+4\ [X]}}{2} \qquad (\ )$$

1,

$$[X]>0 \quad (X<0)>0 \quad\Longrightarrow\quad X\succ\delta_{[X]} \qquad (\ )$$

$\mu_X^+ > E[X] > 0$ $X\sim\delta_\alpha$, . ( ) $\alpha > E[X]$.

$\succeq$ $-1<s,\ t<0$, λ_0

$f(-1,\lambda_0)=s$ $f(t,\lambda_0)=0$. $z\geqslant t$

$$\left[\left(z,\frac{1+t}{1+z};-1,\frac{z-t}{1+z}\right)\right]=t, \qquad \left(z,\frac{1+t}{1+z};-1,\frac{z-t}{1+z}\right)\sim(t,1)$$

$$\left(f(z,\lambda_0),\frac{1+t}{1+z};s,\frac{z-t}{1+z}\right)\sim(0,1) \qquad (10)$$

) / 17.5320.4 0 ((10))()171-371 1 (12)64 0 1 (() 626 0 0 . 626 72.246 465.70 ( ) / 0.0 / 0

Proposition 4. *If the preference relation* $\succeq$ *satisfies distribution regret with a commutative regret function , then it is consistent.*

Proof. $d \in \Re$ $(x, x) = d$ $x \in \mathcal{D}$. (4 d .) $f(x, \lambda) = y$ $(x, y) = d - \lambda$. , x λ,

$$\big(x, f(x, \lambda)\big) = d - \lambda \tag{13}$$

$X \succeq Y$, , $V(\Psi(X, c_Y)) \geqslant 0$. (13)

$$\big(c_X, f(c_X, \lambda)\big) = \big(c_Y, f(c_Y, \lambda)\big) = d - \lambda$$

(12) $(c_X, c_Y) = (f(c_X, \lambda), f(c_Y, \lambda))$,

$$\Psi(\delta_{c_X}, c_Y) = \Psi\big(\delta_{f(c_X,\lambda)}, f(c_Y, \lambda)\big)$$

, $\delta_{c_X} \succeq \delta_{c_Y}$, , $\delta_{f(c_X,\lambda)} \succeq \delta_{f(c_Y,\lambda)}$.
, (13) (12)

$$(x_i, c_X) = \big(f(x_i, \lambda), f(c_X, \lambda)\big)$$

, $\Psi(X, c$

5. Discussion

only [illegible] [10] [illegible] [14].[13]