

## ALGEBRA QUALIFYING EXAM SYLLABUS 2015

### Recommended Textbooks

Abstract Algebra by Dummit and Foote  
Introduction to Commutative Algebra by Atiyah and Macdonald  
Algebra by Lang.

### Topics covered

- I. Groups.
  - Group actions, orbit-stabilizer theorem, Sylow theorems, semi-direct products, Jordan-Holder theorem.
  - Examples: symmetric, alternating, dihedral groups, general and special linear groups.
- II. Linear algebra.
  - Modules over a PID, elementary divisor theorem.
  - Invariant factors and similarity classes of matrices.
  - Jordan and rational canonical forms, Cayley-Hamilton theorem.
- III. Fields.
  - Polynomial rings, Gauss lemma, Eisenstein criterion.
  - Finite fields: construction, classification, structure of the units.
  - Normal and separable extensions, Galois groups and the Galois correspondence.
  - Computing Galois groups of low degree extensions, cyclotomic fields.
  - Discriminants, symmetric polynomials, insolvability of the general quintic.
  - Transcendence degree.
- IV. Rings and commutative algebra.
  - Noetherian and Artinian rings and modules.
  - Discrete valuation rings, local rings, localization, Nakayama's lemma.
  - Primary decomposition.
  - Integral extensions. Going-up and going-down theorems.
- V. Modules and homological algebra.
  - Tensor product of modules and algebras.
  - Exact sequences. Projective, injective, flat modules.
  - Complexes. Projective and injective resolutions, Ext and Tor.
  - Localization of modules.
- VI. Algebraic Geometry.
  - Zariski topology, Spec of a commutative ring, algebraic sets in affine space.
  - Hilbert's Nullstellensatz, Noether Normalization, Krull dimension.
- VII. Algebraic Number Theory.
  - Algebraic integers, discriminants.
  - Prime factorization in Dedekind rings.