Real Analysis Qualifying Exam

Answer all four questions. In your proofs, you may use any major theorem, except the result you are trying to prove (or a variant of it). State clearly what theorems you use. All four questions are worth the same number of points. Good luck.

Notation: In the questions below,

Question 4. Denote by A the smallest algebra of subsets of R that contains all bounded intervals. Denote by A the collection of countable unions of sets in A. Denote by ^{1} the outer measure on the power set P(R) induced by the premeasure on A that assigns to any bounded interval its Euclidean length, and to any unbounded interval 1 .

a. Let E R. What does \E is ^{1} -measurable" (i.e. outer measurable) mean?

b. How is the collection of ^{1}-measurable sets related to the collection of ^{1} -measurable sets?

c. Prove that for any E R and any > 0, there exists A 2 A with E A and ^{1} (A) ^{1} (E) + .