ALGEBRA QUALIFYING EXAM SPRING 2018

Exercise 1. Suppose p is a prime. Show that the Galois group of $x^5 - 1 \in \mathbb{F}_p[x]$ depends only on $p \pmod 5$, and compute it for each congruence class of $p \pmod 5$.

Exercise 2. Let R be a Dedekind domain with field of fractions K Show that for any two proper fractional ideals $I; J$ there are $\alpha; \beta \in K$ with $\alpha I; \beta J \subset R$ integral and $\alpha I + \beta J = R$.

Exercise 3. Suppose that R is a Noetherian ring and $\mathfrak{p} \subset R$ is a prime ideal such that $R_{\mathfrak{p}}$ is an integral domain. Show that there is an $f \in R \setminus \mathfrak{p}$ such that R_f is an integral domain where $R_f = S^{-1}R$ with $S = \{1$