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Abstract

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1 Introduction

2 Literature review

leverage effect

between inventories and prices

leverage *crude oil spot market*

3.1 Descriptive statistics

Q

4 Estimation method

Realized Variance

Integrated Variance

t

$[t \quad k; t]$

$$r(t; k) = \ln F$$

$$\begin{aligned}
 dp &= d \ln(F) \\
 &= \frac{p}{V} dW_1 + x d\text{Poisson}(t) \\
 dV &= (V) dt + \frac{p}{V} dW_2
 \end{aligned}$$

$$E(dW_1 dW_2) = dt$$

$$x \sim N(0, \dots)$$

leverage effect

inverse leverage effect

4.4 Stochastic Volatility model (SV)

$= 0; \quad = 0 \quad = 0)$

e_1
 e_2

J



GMM estimates for the SV, SVJ, SVL, SVJL models for the S&P500 futures: 09/2001–06/2016

	1.507*** (5:87)	0.0869*** (4:58)	0.0424*** (2:92)	0.227*** (29:99)
	0.00398*** (8:41)	0.00994*** (12:56)	0.00649*** (5:55)	0.00376*** (15:69)
	0.283*** (6:55)	0.338*** (19:62)	0.249*** (17:96)	0.12015172*** (54:75)
		0.979 (0:38)		0.156923006*** (10:09)
		0.0159 (0:77)		0.038120618*** (30:60)
			0.379*** (11:29)	0.490*** (29:11)
N	3708			

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

GMM estimates for the SV, SVJ, SVL, SVJL models for Natural Gas futures: 09/2001–06/2016

	0.923** (2:19)	0.772*** (4:11)	0.760*** (3:45)	0.0556* (1:75)
	0.0483*** (4:36)	0.0568*** (6:15)	0.0460*** (5:60)	0.0545*** (4:97)
	1.139** (2:33)	1.041*** (6:23)	0.925*** (3:49)	0.24293*** (3:82)
		0.0101*** (4:03)		0.04345*** (18:52)
		0.932*** (32:63)		0.97814 (0:53)
			0.201*** (4:57)	0.0495** (2:14)
N	3708			

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

GMM estimates for the SV, SVJ, SVL models for WTI futures:
09/2001–06/2016

	0:117 (1:43)	0:0596* (1:77)	0:0963* (1:71)
	0:0247*** (5:75)	0:0224*** (3:45)	0:0242*** (6:43)
	0:176** (2:04)	0:131*** (2:60)	0:162** (2:50)
		0:0190** (2:44)	
		0:439*** (39:24)	
			0:276*** (3:64)
<i>N</i>	3708		

* $p < 0:10$; ** $p < 0:05$; *** $p < 0:01$

5 Robustness check for subsamples

GMM estimates for S&P500, Natural Gas and WTI futures before and after September 15, 2008 (Lehman Brothers bankruptcy)

<i>N</i> = 1699			
	0:137*** (13:27)	0:0871*** (5:55)	0:276*** (3:87)
	0:00331*** (16:47)	0:0836*** (11:69)	0:0328*** (8:33)
	0:0577*** (57:58)	0:5455*** (12:32)	0:343*** (6:36)
	0:1325*** (7:37)	0:0966*** (26:15)	
	0:0364*** (24:70)	0:4921*** (48:64)	
	0:440*** (18:40)	0:0137** (2:20)	0:262*** (6:66)
<i>N</i> = 1990			
	0:188*** (13:22)	0:0434 (1:46)	

6 Out-of-sample performance

0:08014	0:19278	<u>0:07757</u>	0:1104
0:26398	0:28523	<u>0:26231</u>	
0:34547	0:3652	<u>0:33651</u>	0:378
0:06134	0:12978	<u>0:05884</u>	0:07281
0:20573	0:22121	<u>0:20444</u>	
0:26411	0:28076	<u>0:25676</u>	0:27043

0:00447	0:013848	0:004074	<u>0:003308</u>
0:021026	<u>0:019163</u>	0:020501	
0:066412	0:072497	0:057729	<u>0:03341</u>
0:003215	0:005822	0:002886	<u>0:002195</u>
0:015708	<u>0:014377</u>	0:015325	
0:041425	0:046088	0:037161	<u>0:023585</u>

6.2 Diebold–Mariano test

4:80	0:03	5:20	0:39	6:07	0:81	17:64	0:96
0:66	0:42	0:97	0:97	1:31	1:00	12:81	1:00
0:09	0:76	21:50	0:00	31:89	0:00	5:59	1:00
0:05	0:83	27:44	0:00	36:50	0:00	0:67	1:00
0:38	0:54	6:07	0:30	7:38	0:69	24:71	0:74
0:00	0:96	1:65	0:90	2:21	0:99	9:59	1:00
0:17	0:68	5:11	0:40	7:67	0:66	23:01	0:81
0:27	0:61	4:30	0:51	5:96	0:82	22:44	0:84
0:01	0:93	0:04	1:00	0:10	1:00	20:19	0:91
0:01	0:93	0:04	1:00	0:10	1:00	18:55	0:95
0:01	0:94	0:04	1:00	0:10	1:00	22:97	0:82
0:01	0:93	0:04	1:00	0:10	1:00	19:40	0:93
0:05	0:83	0:06	1:00	0:25	1:00	19:37	0:93
0:01	0:92	0:05	1:00	0:11	1:00	13:84	0:99
6:68	0:01	7:15	0:21	11:43	0:32	22:10	0:85
4:05	0:04	4:37	0:50	4:29	0:93	4:76	1:00
0:00	1:00	1:91	0:86	4:71	0:91	17:86	0:96
0:15	0:69	0:76	0:98	1:52	1:00	15:42	0:99
0:00	0:98	2:05	0:84	4:67	0:91	17:12	0:97
0:10	0:75	0:65	0:99	1:30	1:00	10:38	1:00
0:00	1:00	2:06	0:84	4:75	0:91	17:22	0:97
0:11	0:74	0:75	0:98	1:60	1:00	10:23	1:00

0:094	0:231	3:253	0:001	65:850	0:006
0:303	0:000	7:853	0:000	37:832	0:568
0:296	0:000	8:172	0:000	40:898	0:431
0:452	0:000	9:018	0:000	34:394	0:720
0:068	0:626	0:928	0:177	85:452	0:000
0:259	0:000	7:420	0:000	43:723	0:316
0:072	0:554	0:799	0:212	87:346	0:000
0:249	0:000	7:090	0:000	39:267	0:503
0:311	0:000	8:799	0:000	6:095	1:000
0:483	0:000	9:252	0:000	0:421	1:000
0:314	0:000	8:809	0:000	5:425	1:000
0:494	0:000	9:254	0:000	0:397	1:000
0:151	0:007	7:048	0:000	20:472	0:996
0:431	0:000	9:126	0:000	1:917	1:000
0:052	0:888	2:377	0:009	34:724	0:706
0:303	0:000	8:145	0:000	21:069	0:994
0:046	0:958	1:521	0:064	42:030	0:383
0:276	0:000	7:644	0:000	34:788	0:704
0276	0				

0	0	0	0	0	0
39	31	70	196	223	166
311	319	280	154	127	184
350	350	350	350	350	350

0	0	0	0	0	0
336	45	60	165	69	304
14	305	290	185	281	46
350	350	350	350	350	350

0	0	0	0
71	8	258	199
279	342	92	151
350	350	350	350

7 Forecasting VaR and CVaR

b

Stage 1 *Backtesting the VaR and CVaR models*

Failure Rate FR violation rate

= 5% = 1%

FRVaR

FRVaR

CVaR

FRVaR *FRVaR*

$$FRVaR = \frac{1}{T} \sum_{t=1}^T I(y_t < VaR)$$

$$FRVaR = \frac{1}{T} \sum_{t=1}^T I(y_t > VaR)$$

VaR

VaR

VaRs

T

I()

$$\text{Downside: } I = \begin{cases} 1 & \text{if } y < VaR \\ 0 & \text{if } y \geq VaR \end{cases}$$

$$\text{Upside: } I = \begin{cases} 1 & \text{if } y > VaR \\ 0 & \text{if } y \leq VaR \end{cases}$$

LR

$H_0 : FR =$

LR

LR

LR

LR

b

LR

Out of sample VaR backtesting results using Simulated Volatilities at different risk levels

		VaR		LR		p values		LR					
	LR			LR	LR	LR	LR	LR	LR				
5%	VaR	4:06%	7:32%	13:0%	0:69	0:24	0*	0:62	0:27	0*	0:51	0:23	0:93
	VaR	7:32%	7:32%	14:63%	0:03*	0:24	0*	0:27	0:27	0*	0:6	0:23	0:02*
1%	VaR	0:81%	0:81%	6:50%	0:96	0:96	0*	0:83	0:83	0*	0:9	0:9	0:52
	VaR	3:25%	4:06%	6:50%	0*	0:03	0*	0:05	0:01	0*	0:06	0:51	0:45
5%	VaR	13:82%	8:13%	8:13%	0*	0:14	0:14	0:01*	0:18	0:83	0*	0:13	0:30
	VaR	18:67%	9:76%	11:382%	0*	0:03*	0:01*	0:61	0:10	0:73	0*	0:02*	0:02*
1%	VaR	8:94%	0:81%	3:252%	0*	0:83	0:05	0:01*	0:9	0:60	0*	0:96	0:12
	VaR	13:01%	5:69%	4:878%	0*	0*	0*	0:13	0:35	0:27	0*	0*	0*
5%	VaR	2:44%	7:32%	8:94%	0:32	0:24	0:17	0:15	0:27	0:07	0:7	0:23	0:99
	VaR	4:88%	8:13%	11:38%	0:02*	0:13	0:01*	0:95	0:14	0*	0:21	0:18	0:63
1%	VaR	0:81%	0:81%	5:69%	0:96	0:96	0*	0:83	0:83	0*	0:9	0:9	0:39
	VaR	4:06%	4:06%	4:06%	0*	0:03	0:03	0:01	0:01	0:01	0:13	0:51	0:51
5%	VaR	2:44%		4:88%	0:32		0:7	0:15		0:95	0:6973		0:43
	VaR	8:13%		6:50%	0:02*		0:40	0:14		0:4636	0:75		0:29
1%	VaR	0:81%		1:63%	0:96		0:77	0:83		0:5223	0:9		0:8
	VaR	2:44%		1:63%	0:01*		0:77	0:17		0:5223	0:75		0:8

Out of sample CVaR backtesting results using Simulated Volatilities at different nominal risk levels b

LR

CVaR

LR p values

LR p values

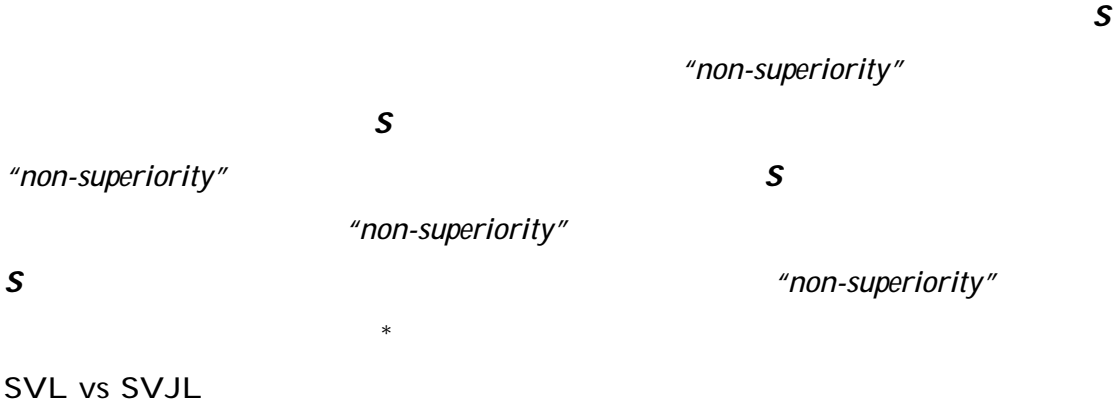
		LR				LR				LR			
		p values				p values				p values			
1:96%	CVaR	1:6%	0:8%	11:4%	0:91	0:57	0*	0:78	0:3	0*	0:8	0:9	0:57
	CVaR	3:2%	4:1%	9:8%	0*	0:26	0*	0:34	0:14	0*	0:07	0:5	0:93
0:38%	CVaR	0:8%	0:8%	5:7%	0:78	0:78	0*	0:5	0:5	0*	0:9	0:9	0:39
	CVaR	2:4%	2:4%	4:9%	0:04	0:042	0*	0:01	0:01	0*	0:7	0:7	0:47
1:96%	CVaR	0:11	0:02	0:04	0*	0:78	0:14	0:03	0:80	0:51	0*	0:92	0:26
	CVaR	0:14	0:06	0:07	0*	0:02*	0*	0:20	0:36	0:68	0*	0:03	0*
0:38%	CVaR	0:07	0:01	0:02	0*	0:50	0:01	0:14	0:90	0:70	0*	0:78	0:04
	CVaR	0:12	0:04	0:05	0*	0*	0*	0:10	0:51	0:27	0*	0:00	0*
1:96%	CVaR	0:8%	0:8%	6:5%	0:6	0:57	0*	0:3	0:3	0*	0:9	0:9	0:5
	CVaR	4:1%	4:1%	7:3%	0*	0:26	0*	0:14	0:14	0*	0:13	0:51	0:26
0:38%	CVaR	0%	0:8%	3:2%	NA	0:78	0*	0:33	0:5	0*	NA	0:9	0:1
	CVaR	1:6%	2:4%	4:1%	0:23	0:04	0*	0:1	0:01	0*	0:8	0:7	0:5
1:96%	CVaR	1:6%											

CVaR 3: :026

:

CVaR 1: 1:6%

Stage 2 Efficiency measures.



SVL vs SVJ

Out of sample RLF and FLF loss function approach applied to the models surviving the VaR Backtesting stage.

	5%	1%	5%	1%
	<u>0</u>	<u>0:03</u>	<u>0:02</u>	<u>0:01</u>
	0	0	0:01	0
		<u>0:07</u>	<u>0:11</u>	<u>0:01</u>
		0	0:01	0
			<u>0:02</u>	<u>0:01</u>
			0	0
			<u>0:01</u>	<u>0:01</u>

Out of sample RLF and FLF loss function approach applied to the models surviving the CVaR Backtesting stage

References

Appendix I: Realized Variance and Moment conditions

Realized Variance
Integrated Vari-
ance

8.1 No jumps

$$r(t; k) = \ln F_t - \ln F_{t-k} = \int_{t-k}^t (\cdot) d + \int_{t-k}^t (\cdot) dW$$

$$QV(t; k) = IV(t; k) = \int_{t-k}^t (\cdot)^2 d$$

Realized Variance

$$RV(t; k; n) = \sum_{j=1}^n r(t; k + \frac{j}{n}; \frac{1}{n})^2$$

$$RV(t; k; n) \rightarrow IV(t; k)$$

$$n \rightarrow \infty$$

$$I = 1$$

8.2 Jumps

t $[t, k; t]$

$$r(t; k) = \frac{\ln F - \ln F_{-Z}}{Z} = \frac{(\quad) dt + (\quad) dW + x(\quad) dN(\quad)}{Z}$$

$$e_1 = E[BP_{+1j}G] + \sigma^2 dt RV_{+1}$$

$$E[RV_{+1j}G] = E[BP_{+1j}G] + \sigma^2 dt$$

(A:3) *Appendix A.1*

Residual 2

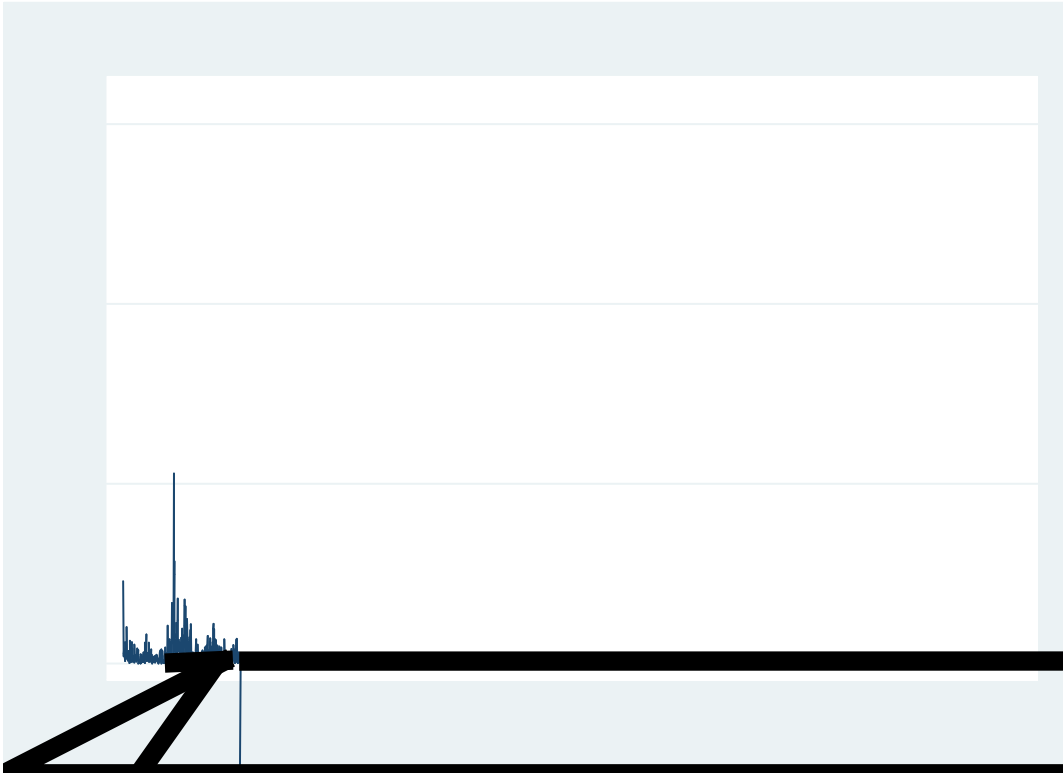
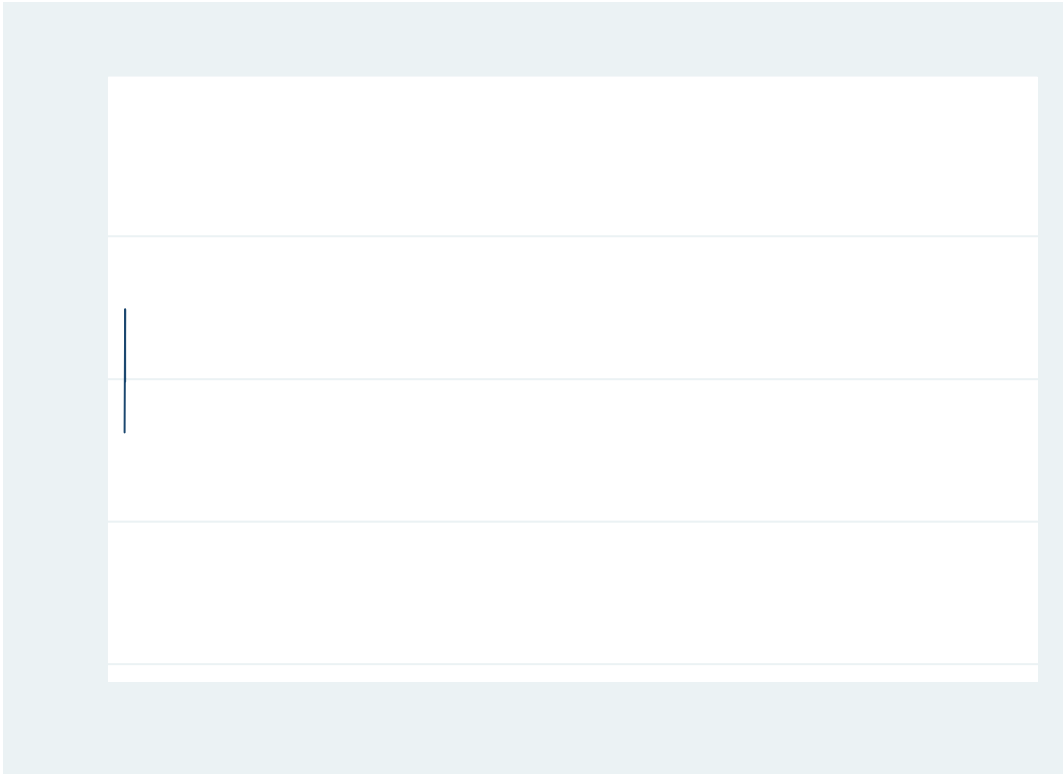
$(t+1; t+2)$

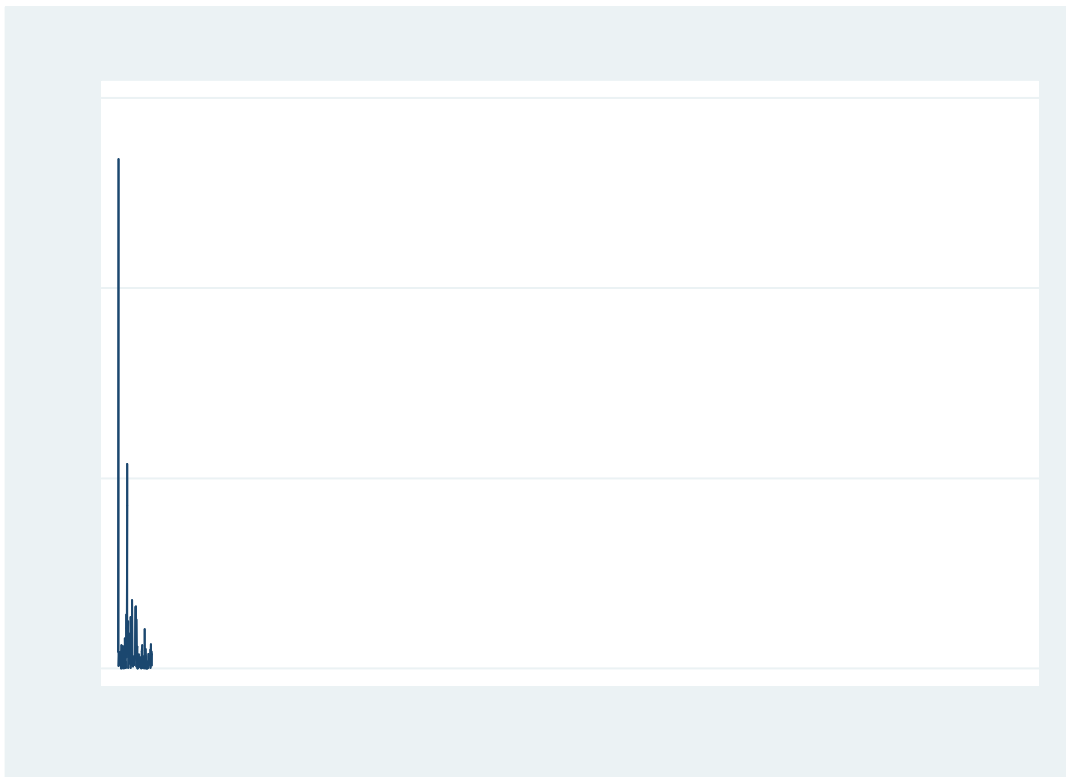
$$e_2 = E[RV_{+1}$$

$$E V^2_{+1} G$$

$$V^2_{+1 +2}$$

Appendix II: Figures





Appendix III: t and J tests on the moment conditions

GMM estimates for SV model for the S&P500 futures: 09/2001–06/2016

$E[V_{+1} +2jG]$	$V_{+1} +2$	0:0052	0:00271	0:997841
$E[V_{+1} +2V_{-16} -15jG]$	$V_{+1} +2V_{-16} -15$	0:0052	0:00844	0:993267
$E[V_{+1} +2V_{-17} -16jG]$	$V_{+1} +2V_{-17} -16$	0:0052	0:05157	0:958872
$E[V_{+1} +2G]$	$V_{+1} +2$	0:00851	0:00735	0:994135
$E[V_{+1} +2V_{-10} -9G]$	$V_{+1} +2V_{-10} -9$	0:00851	0:01141	0:9909
$E[V_{+1} +2V_{-}]$				

GMM estimates for SVJ model for the S&P500 futures: 09/2001–06/2016

$E[V_{+1} +2jG]$	$V_{+1} +2$	0:000068	0:999994
$E[V_{+1} +2V_{-16} -15jG]$	$V_{+1} +2V_{-16} -15$	0:000068	0:999995
$E[V_{+1} +2V_{-17} -16jG]$	$V_{+1} +2V_{-17} -16$	0:000068	0:999950
$E[V_{+1} +2G]$	$V_{+1} +2$	0:000865	0:999991
$E[V_{+1} +2V_{-10} -9G]$	$V_{+1} +2V_{-10} -9$	0:000865	0:999930
$E[V_{+1} +2V_{-18} -17G]$	$V_{+1} +2V_{-18} -17$	0:000865	0:999938
$E[V_{+1} +2V_{-18} -17G]$	$V_{+1} +2V_{-18} -17$	0:002765	0:999498
$E[V_{+1} +2V_{-16} -15G]$	$V_{+1} +2V_{-16} -15$	0:002765	0:999712
$E[V_{+1} +2V_{-2} -1G]$	$V_{+1} +2V_{-2} -1$	0:002765	0:993492
$E[V_{+1} +2G]$	$V_{+1} +2$	0:000559	0:999998
$E[V_{+1} +2V_{-10} -9G]$	$V_{+1} +2V_{-10} -9$	0:000559	0:999986
$E[V_{+1} +2V_{-18} -17G]$	$V_{+1} +2V_{-18} -17$	0:000559	0:999803
J	J	$\frac{2}{J} = 6:96664$	0:4324
P	J		

GMM estimates for SVL model for the S&P500 futures: 09/2001–06/2016

$E[V_{+1} +2jG]$	$V_{+1} +2$	0:0001	0	1
$E[V_{+1} +2V_{-16} -15jG]$	$V_{+1} +2V_{-16} -15$	0:0001	0	1
$E[V_{+1} +2V_{-17} -16jG]$	$V_{+1} +2V_{-17} -16$	0:0001	0	1
$E[V_{+1} +2V_{-20} -19jG]$	$V_{+1} +2V_{-20} -19$	0:0001	0:000025	0:99998
$E[V_{+1} +2G]$	$V_{+1} +2$	0:000232	0	1
$E[V_{+1} +2V_{-20} -19G]$	$V_{+1} +2V_{-20} -19$	0:000232	0	1
$E[V_{+1} +2V_{-18} -17G]$	$V_{+1} +2V_{-18} -17$	0:000232	0	1
$E[V_{+1} +2V_{-22} -21G]$	$V_{+1} +2V_{-22} -21$	0:000232	0:000025	0:99998
$E[p_{+1}V_{+1} +2jG]$	$p_{+1}V_{+1} +2$	0:00023	0	1
$E[p_{+1}V_{+1} +2p_{-15} -14V_{-15} -14G]$	$p_{+1}V_{+1} +2p_{-15} -14V_{-15} -14$	0:00023	0	1
$E[p_{+1}V_{+1} +2p_{-12} -11V_{-12} -11G]$	$p_{+1}V_{+1} +2p_{-12} -11V_{-12} -11$	0:00023	0	1
$E[p_{+1}V_{+1} +2p_{-18} -17V_{-18} -17G]$	$p_{+1}V_{+1} +2p_{-18} -17V_{-18} -17$	0:00023	2:9E 05	0:999977
t	J	2	$11:177$	$0:1919$
p	J			

GMM estimates for SVJL model for the S&P500 futures: 09/2001–06/2016

$E[V_{+1} +2jG]$	$V_{+1} +2$	2:2E	05	0	1
$E[V_{+1} +2V_{-20}^2 -19G]$	$V_{+1} +2V_{-20}^2 -19G$	2:2E	05	0	1
$E[V_{+1} +2V_{-18}^3 -17G]$	$V_{+1} +2V_{-18}^3 -17G$	2:2E	05	2E	0:999998
$E[V_{+1} +2G]$	$V_{+1} +2$	3E	06	0	1
$E[V_{+1} +2V_{-10}^2 -9G]$	$V_{+1} +2V_{-10}^2 -9G$	3E	06	0	1
$E[V_{+1} +2V_{-25}^4 -24G]$	$V_{+1} +2V_{-25}^4 -24G$	3E	06	0	1
$E[p_{+1}V_{+1} +2jG]$	$p_{+1}V_{+1} +2$	3E	06	0	1
$E[p_{+1}V_{+1} +2p_{-15} -14V_{-15} -14jG]$	$p_{+1}V_{+1} +2p_{-15} -14V_{-15} -14jG]$	3E	06	0	1
$p_{+1}V_{+1} +2p_{-15} -14V_{-15} -14$					
$E[p_{+1}V_{+1} +2p_{-12} -11V_{-12} -11jG]$	$p_{+1}V_{+1} +2p_{-12} -11V_{-12} -11jG]$	3E	06	0	1
$p_{+1}V_{+1} +2p_{-12} -11V_{-12} -11$					
$E[V_{+1}jG]$	V_{+1}	0:000021		0	1
$E[V_{+1}V_{-16} -15G]$	$V_{+1}V_{-16} -15G$	0:000021		0	1
$E[V_{+1}V_{-22} -21G]$	$V_{+1}V_{-22} -21G$	0:000021		0	1
$E[V_{+1}^2G]$	V_{+1}^2G	3E	06	0	1
$E[V_{+1}^2V_{-10} -9G]$	$V_{+1}^2V_{-10} -9G$	3E	06	0	1
$E[V_{+1}^2V_{-18} -17G]$	$V_{+1}^2V_{-18} -17G$	3E	06V ₊₁	0	t 2 1
			+1	10 ^v	1

GMM estimates for SVJ model for the Natural Gas futures: 09/2001–06/2016

$E[V_{+1+2}jG]$	V_{+1+2}	0:005914	0:999994
$E[V_{+1+2}V_{-16-15}jG]$	$V_{+1+2}V_{-16-15}$	0:005914	0:999995
$E[V_{+1+2}V_{-17-16}jG]$	$V_{+1+2}V_{-17-16}$	0:005914	0:999950
$E[V_{+1+2}^2G]$	V_{+1+2}^2	0:009844	0:999991
$E[V_{+1+2}^2V_{-10-9}^2G]$	$V_{+1+2}^2V_{-10-9}^2$	0:009844	0:999930
$E[V_{+1+2}^2V_{-18-17}^2G]$	$V_{+1+2}^2V_{-18-17}^2$	0:009844	0:999938
$E[V_{+1j}G]$	V_{+1}	0:002155	0:999498
$E[V_{+1}BP_{-16-15}^2G]$	$V_{+1}BP_{-16-15}^2$	0:002155	0:999712
$E[V_{+1}BP_{-2-1}^2G]$	$V_{+1}BP_{-2-1}^2$	0:002155	0:993492
$E[V_{+1}^2G]$	V_{+1}^2	0:005225	0:999998
$E[V_{+1}^2V_{-10-9}^2G]$	$V_{+1}^2V_{-10-9}^2$	0:005225	0:999986
$E[V_{+1}^2BP_{-18-17}^2G]$	$V_{+1}^2BP_{-18-17}^2$	0:005225	0:999803
$\hat{\gamma}$		5:81155	0:5619
ρ			

GMM estimates for SVL model for the Natural Gas futures: 09/2001–06/2016

$E[V_{+1} + 2jG]$	$V_{+1} + 2$	0:0058	0:00262	0:997913
$E[V_{+1} + 2V_{-16} - 15jG]$	$V_{+1} + 2V_{-16} - 15$	0:0058	0:00277	0:997792
$E[V_{+1} + 2V_{-17} - 16jG]$	$V_{+1} + 2V_{-17} - 16$	0:0058	0:0678	0:945947
$E[V_{+1} + 2G]$	$V_{+1} + 2$	0:01161	0:01107	0:991166
$E[V_{+1} + 2V_{-10} - 9G]$	$V_{+1} + 2V_{-10} - 9$	0:01161	0:0099	0:992101
$E[V_{+1} + 2V_{-18} - 17G]$	$V_{+1} + 2V_{-18} - 17$	0:01161	0:18512	0:853143
$E[p_{+1}V_{+1} + 2jG]$	$p_{+1}V_{+1} + 2$	0:00281	0:00116	0:999072
$E[p_{+1}V_{+1} + 2p_{-15} - 14V_{-15} - 14jG]$	$p_{+1}V_{+1} + 2p_{-15} - 14V_{-15} - 14jG$	0:00281	0:00544	0:995658
$E[p_{+1}V_{+1} + 2p_{-18} - 17V_{-18} - 17jG]$	$p_{+1}V_{+1} + 2p_{-18} - 17V_{-18} - 17jG$	0:00281	0:02369	0:9811

GMM estimates for SVJL model for the Natural Gas futures: 09/2001–06/2016

$E[V_{+1} +2jG]$	$V_{+1} +2$	0:00061	0:00029	0:999771
$E[V_{+1} +2V_{-10} -9jG]$	$V_{+1} +2V_{-10} -9$	0:00061	0:00194	0:998452
$E[V_{+1} +2V_{-18} -17G]$	$V_{+1} +2V_{-18} -17$	0:00061	0:00024	0:999809
$E[V_{+1} +2G]$	$V_{+1} +2$	0:00602	0:00574	0:995417
$E[V_{+1} +2V_{-10} -9G]$	$V_{+1} +2V_{-10} -9$	0:00602	0:14815	0:882232
$E[V_{+1} +2V_{-25} -24G]$	$V_{+1} +2V_{-25} -24$	0:00602	0:01217	0:99029
$E[p_{+1}V_{+1} +2jG]$	$p_{+1}V_{+1} +2$	0:000105	0:00003	0:999976
$E[p_{+1}V_{+1} +2p_{-15} -14V_{-15} -14jG]$	$p_{+1}V_{+1} +2p_{-15} -14V_{-15} -14jG]$	0:000105	0:000015	0:999988
$p_{+1}V_{+1} +2p_{-15} -14V_{-15} -14$				
$E[p_{+1}V_{+1} +2p_{-12} -11V_{-12} -11jG]$	$p_{+1}V_{+1} +2p_{-12} -11V_{-12} -11jG]$	0:000105	0:000264	0:999789
$p_{+1}V_{+1} +2p_{-12} -11V_{-12} -11$				
$E[V_{+1}jG]$	V_{+1}	0:052523	0:057867	0:953858
$E[V_{+1}V_{-6} -5G]$	$V_{+1}V_{-6} -5$	0:052523	4:326995	0:000016
$E[V_{+1}V_{-2} -1G]$	$V_{+1}V_{-2} -1$	0:052523	0:344424	0:730547
$E[V_{+1}G]$	V_{+1}	0:00536	0:00157	0:998747
$E[V_{+1}V_{-10} -9G]$	$V_{+1}V_{-10} -9$	0:00536	0:00151	0:998795
$E[V_{+1}V_{-18} -17G]$	$V_{+1}V_{-18} -17$	0:00536	0:00746	0:994051
J	J	$\frac{2}{9}$	$\frac{6}{9} = 6:00869$	$\frac{0}{9} = 0:7390$
P	J			

GMM estimates for SV model for the WTI futures: 09/2001–06/2016

$E[V_{+1} \ +2jG]$	$V_{+1} \ +2$	0:000451	0:000079	0:999937
$E[V_{+1} \ +2V_{-16} \ -15jG]$	$V_{+1} \ +2V_{-16} \ -15$	0:000451	0:000044	0:999965
$E[V_{+1} \ +2V_{-17} \ -16jG]$	$V_{+1} \ +2V_{-17} \ -16$	0:000451	0:000586	0:999532
$E \ V$				

GMM estimates for SVL model for the WTI futures: 09/2001–06/2016

$E[V_{+1} + 2jG]$	$V_{+1} + 2$	0:000451	0:000017	0:999987
$E[V_{+1} + 2V_{-19} - 18jG]$	$V_{+1} + 2V_{-19} - 18$	0:000451	0:00002	0:999984
$E[V_{+1} + 2V_{-17} - 16jG]$	$V_{+1} + 2V_{-17} - 16$	0:000451	0:000548	0:999563
$E[V_{+1} + 2G]$	$V_{+1} + 2$	0:001539	0:000007	0:999994
$E[V_{+1} + 2V_{-10} - 9G]$	$V_{+1} + 2V_{-10} - 9$	0:001539	0:000024	0:999981
$E[V_{+1} + 2V_{-18} - 17G]$	$V_{+1} + 2V_{-18} - 17$	0:001539	0:001058	0:999156
$E[p_{+1}V_{+1} + 2jG]$	$p_{+1}V_{+1} + 2$	0:000108	0:000006	0:999995
$E[p_{+1}V_{+1} + 2p_{-18} - 17V_{-18} - 17jG]$	$p_{+1}V_{+1} + 2p_{-18} - 17V_{-18} - 17jG$	0:000108	0:000001	0:999999
$E[p_{+1}V_{+1} + 2p_{-16} - 15V_{-16} - 15jG]$	$p_{+1}V_{+1} + 2p_{-16} - 15V_{-16} - 15jG$	0:000108	0:000097	0:999923

t	J	p	J
	2	5	1:37354
			0:9272