The Demand for Divisia Money: Theory and Evidence

Michael T. Belongia
Otho Smith Professor of Economics
University of Mississippi
Box 1848
University, MS 38677
mvpt@earthlink.net

Peter N. Ireland
Department of Economics
Boston College
140 Commonwealth Avenue
Chestnut Hill, MA 02467
peter.ireland@bc.edu

November 2017

Abstract: A money-in-the-utility function model is extended to capture the distinct roles of noninterest-earning currency and interest-earning deposits in providing liquidity services to households. It implies the existence of a stable money demand relationship that links a Divisia monetary aggregate to spending or income as a scale variable and the associated Divisia user-cost dual as an opportunity cost measure. Cointegrating money demand equations of this form appear in quarterly United States data spanning the period from 1967:1 through 2017:2, especially for the Divisia M2 aggregate. The identification of a stable money demand function over a period that includes the financial innovations of the 1980s and continues through the recent financial crisis and Great Recession suggests that a properly measured aggregate quantity of money can play a role in the conduct of monetary policy. That role can be of greater prominence when traditional interest rate policies are constrained by the zero lower bound.

JEL Codes: C43, E41.

Keywords: Divisia monetary aggregates, money demand, money-in-the-utility function.

Acknowledgments: Neither of us received any external support for, or hA

Introduction

Since Milton Friedman's (1956) "restatement," the existence of a stable money demand function has been regarded as a necessary pre-condition for the success of any quantity-theoretic approach to monetary policy that would use information in broad monetary aggregates to achieve goals for aggregate spending or the price level. The general message of Friedman's essay, supported by the empirical papers that accompanied its publication, motivated a broad and active line of research in monetary economics that lasted more than three decades.[1] The primary focus of this agenda was the search for money, he now (Taylor 1993) suggested that Federal Reserve policy, beginning in the late 1980s, could be adequately described by a strikingly simple by introducing separate roles for noninterest-earning currency and interest-earning deposits in providing a representative household with liquidity services that allow it to purchase goods and services at the expense of less time and effort. This extension to the theory makes clear that the money demand relationship implied by the model applies to a Divisia monetary aggregate but not a simple-sum measure of the type provided officially by the Federal Reserve. The same theoretical extension also reinforces Belongia's (2006) argument that the price, or user-cost, dual to the Divisia monetary aggregate ought to appear in place of a short-term nominal interest rate as a preferred opportunity cost measure in the money demand equation. Finally, the extended money-in-the-utility function model can be used to motivate renewed interest in classic empirical specifications for money demand, originally proposed by Cagan (1956), Selden

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opportunity cost measure. Because the theory suggests, however, that the "price" of monetary services is the user cost dual to the economic quantity aggregate, the empirical specifications that follow use this measure instead. The results presented here, therefore, provide evidence of stable money demand relationships based on price and quantity data derived from the same principles.

Theory

The Model

As noted above, the model developed here builds further on Lucas' (2000) variant of the money-in-the-utility function models of Sidrauski (1967) and Brock (1974) by introducing separate roles for noninterest-earning currency and interest-earning deposits in providing a representative household with liquidity services that allow it to purchase goods and services with less effort. Through this extension, the theory makes clear that the aggregate of currency and deposits appearing in a properly-specified money demand relationship is a Divisia aggregate, and not simple sum measures like those constructed, officially, by the Federal Reserve. While the description of household optimization provided here could be incorporated into a dynamic, stochastic, general equilibrium model along the same lines followed by Belongia and Ireland (2014) and Ireland (2014), the perfect foresight, partial equilibrium framework used here, instead, simplifies the analysis by abstracting from unnecessary general equilibrium considerations and highlights, as well, that the basic properties of the money demand relationship derived here do not depend on the details of how other sectors of the economy might be modeled.

Also for simplicity, the theoretical analysis proceeds here under the assumption that a representative household substitutes between currency and a single type of interest-earning bank deposit in its efforts to construct the portfolio of these two liquid assets that most efficiently provides the monetary services it uses in making transactions during each period. Additional types of deposits, all paying interest at different rates and each playing its own role in the household's portfolio of monetary assets, could easily be incorporated, at the cost of requiring slightly more detailed notation and more tedious algebra. The empirical work, by contrast, uses various Divisia monetary aggregates that do include a wide range of monetary assets available in the United States today.

In the model, an infinitely-lived representative household enters each period $t = 0,1,2,..$ with M_{t-1} units of currency and B_{t-1} bonds. At the very start of a beginning-of-period asset trading and allocation session, the household receives T_t additional u household uses some of its currency to purchase

In (2), the function v captures the time-and-effort-saving services provided to the representative household's shopper by the monetary aggregate M_t^a, formed from currency and deposits according to

$$g(N_t, D_t) \quad M_t^a,$$

where the monetary aggregator g is assumed to be homogeneous of degree one, so that the underlying transaction technology exhibits constant returns to scale and so that this constraint can be rewritten equivalently in real terms as

$$g \;\; \frac{N_t}{P_t}, \frac{D_t}{P_t} \qquad \frac{M_t^a}{P_t} \tag{3}$$

for all $t = 0,1,2,...$ As noted by Lucas (2000), the specific form of the utility function in (2) makes the model consistent with balanced growth: if the household's real income y_t grows at a constant long-run growth rate, then its optimal choices of consumption aggregation is inconsistent with economic theory, and simple-sum monetary aggregates are unlikely to measure accurately the true flows of monetary services demanded by households.

Also at the end of each period $t = 0,1,2,\ldots$, the household receives an interest payment $r_t^d D_t$ on the deposits it holds during the period, but must also repay with interest $r_t^l L_t$ the loans it received earlier from the bank. After accounting for these receipts and payments, as well as the income $P_t y_t$ earned and funds $P_t c_t$ spent during the period, the household carries M_t units of currency into period $t+1$, where

$$c_t v \frac{M_t^a}{P_t c_t} \quad v \frac{M_t^a}{P_t c_t} = {}^{1}_{t}, \tag{12}$$

and

$$\frac{{}^{3}_{t}}{P_t} = \frac{{}^{2}_{t+1}}{P_{t+1}}, \tag{13}$$

together with (3), (5), and (6) as equalities for all $t = 0,1,2...$.

<u>Monetary Aggregation and Money Demand</u>

Note that (7) and (13) imply

$${}^{2}_{t} = (1 + r_t) \; {}^{3}_{t}. \tag{14}$$

Comparing (10) and (14) then reveals that the absence of arbitrage requires $r_t = r_t^l$ for all $t = 0,1,2,...$, so that the interest rate on bonds and loans are always equal. This result reflects the fact that in the model, for simplicity, bonds and loans are both risk-free assets that provide one of choosing B_t, L_t, c_t, M_t^a, and M_t for all $t = 0,1,2,...$ to maximize the utility function in (2) subject to the constraints

$$u_t^a M_t^a = u_t^n N_t + u_t^d D_t. \tag{19}$$

The left-hand side of (19) measures total expenditures on monetary services; the right-hand side decomposes these expenditures into components provided by currency and deposits. Although neither of the two terms on the left-hand side, the user cost u_t^a nor the quantity aggregate M_t^a, is observable individually, all of the terms on the right-hand side are observable from data on currency, deposits, and the interest rates on deposits and the benchmark, illiquid asset. Thus, total expenditures on the left-hand side can be inferred from the sum on the right, and the expenditure shares for currency

$$s_t^n = \frac{u_t^n N_t}{u_t^a M_t^a} = \frac{u_t^n N_t}{u_t^n N_t + u_t^d D_t}$$

$$\frac{v\ (m^a)}{v(m^a)} = \frac{(m^a)}{1 + m^a\quad (m^a)}, \tag{23}$$

which coincides with Lucas' (2000, p.257) equation 3.9 except that, again, the functions v and ψ have, as their arguments, the ratio of a real Divisia monetary aggregate to consumption.

Equations (21) and (23) can be used to specialize the model so that it motivates several classical empirical formulations for money demand, modified here to relate the demand for a vU

checkable deposits. Divisia M2 adds savings deposits, including money market deposit accounts, retail money market mutual fund shares, and small time deposits. The MZM monetary aggregate, which excludes the small time deposit component of M2 but adds institutional money market mutual fund shares, was first proposed by Motley (1988) and given the label "money, zero maturity" by Poole (1991). The CFS data include a Divisia measure at this level of aggregation as well. Finally, Divisia M4 – the broadest aggregate compiled by the CFS – combines all of the assets in M2 and MZM with large time deposits, overnight and term repurchase agreements, commercial paper, and United States Treasury bills to obtain a collection similar to that included in the Federal Reserve's discontinued L measure of liquidity.

real demand for money relative to a scale variable on the one hand, and either the log or the

miss. Benati (2017) relates similar findings linking low-frequency movements in simple-sum

income as his scale variable, Meltzer used a simple-sum M2 aggregate together with a long-term corporate bond rate to measure its opportunity cost. If one interprets these measures as error-ridden, relative to the Divisia M2 aggregate and its associated user cost, one would expect the resulting elasticity estimates lower bound, the existence of a st

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Table 1. Cointegrating Money Demand Relationships

Meltzer Specification: $\ln(m_t^a) = {}_0 \quad {}_1\ln(r_t \quad r_t^a)$

$\ln(m_t^a)$: log money -consumption ratio

Sample Period: 1967:1 2017:2

Divisia Aggregate	q	${}_0$	${}_1$	Max Eigenvalue $r=0$	p-value	Max Eigenvalue $r=1$	p-value
M1	2^{*+}	-8.53	2.75	46.36	0.0001	3.79	0.5466
	3	-7.35	2.24	37.35	0.0001	4.11	0.4988
	4	-6.44	1.85	27.69	0.0009	3.28	0.6337
	5	-6.68	1.96	20.79	0.0156	2.62	0.7463
M2	2^{*+}	-4.93	1.79	48.28	0.0001	6.32	0.1979
	3	-4.84	1.72	44.91	0.0001	6.31	0.2145
	4	-4.71	1.63	36.39	0.0001	6.63	0.1858
	5	-4.92	1.77	33.83	0.0002	5.11	0.3157
MZM	2^{*+}	-4.59	1.57	42.23	0.0001	7.01	0.1385
	3	-4.43	1.45	40.37	0.0001	6.87	0.1558
	4	-4.43	1.45	29.54	0.0001	7.25	0.1375
	5	-4.47	1.47	29.65	0.0005	5.71	0.2499
M4	2^{*+}	-9.99	5.82	28.29	0.0005	9.71	0.0427
	3						

Table 2. Cointegrating Money Demand Relationships

Cagan Specification: $\ln(m_t^a) = \ {}_0 \quad {}_1(r_t \quad r_t^a)$

$\ln(m_t^a)$: log money -consumption ratio

Sample Period: 1967:1 2017:2

Divisia Aggregate	q	${}_0$	${}_1$	Max Eigenvalue $r = 0$	p-value	Max Eigenvalue $r = 1$	p-value
M1	2*+	1.81	33.56	32.18	0.0004	3.13	0.6032
63.92Tre.5 93 4201 31. 1.16.21 Tw9.24857690.81 0.05 60 7.84 9.8(28.)4.5 0; 4 n BT 0.5 BT 28 c 46 W W 8 B T 0 4 T 0 1 T w 9 . 5 2 m [(2 0) 0 T0W 4 167.52 m [(28.)4.m (3)Tj 0.63 0							

Table 3. Cointegrating Money Demand Relationships

Selden - LatanØ Specification: $v_t^a = \quad_0 + \quad_1(r_t \quad r_t^a)$

v_t^a : consumption velocity

Sample Period: 1967:1 2017:2

Divisia Aggregate	q	$_0$	$_1$	Max Eigenvalue $r = 0$	Max Eigenvalue

Figure 1. Divisia Monetary Data. The log money-consumption ratio in column one and consumption velocity in column four are computed using nominal personal consumption expenditures as the scale variable.

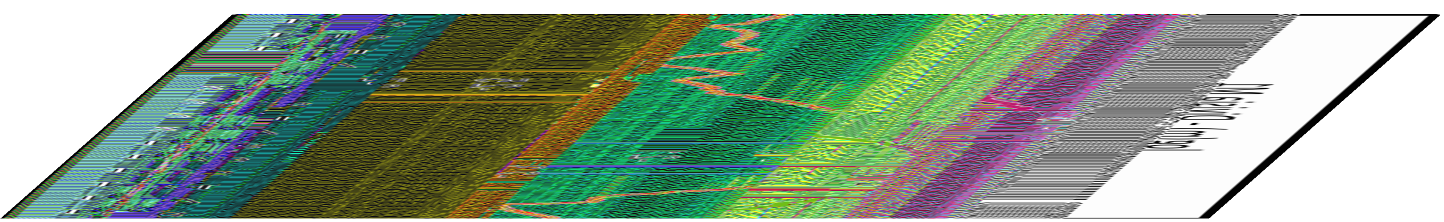

Figure 2. Divisia Money Demand Relationships. Each scatterplot compares the indicated Divisia monetary aggregate, scaled by nominal personal consumption expenditures, measured along the vertical axis, to the associated user cost aggregate, measured along the horizontal axis. The Meltzer (1963) specification relates the log money-consumption ratio to the log of the user cost; the Cagan (1956) specification relates the log money-consumption ratio to the level of the user cost; the Selden-

Table A1. Efficient Unit Root Test Results

$\ln(m_t^a)$: log money -consumption ratio

Sample Period: 1967:1 – 2017:2

Divisia Aggregate		ADF-GLS		ADF-GLS	
					0.0001
	4	-0.66	0.4923	-3.34	0.0010
M2	1	-0.01	0.3783	-5.40	0.0001
	2	-0.11	0.4028	-4.46	0.0002
	3	-0.20	0.4262	-4.13	0.0002
	4	-0.18	0.4334	-3.61	0.0002
MZM	1	-0.19	0.4446	-5.56	0.0001
			0.4601	-3.85	0.0002
	3	0.39	0.3250	-5.24	[redacted]
	4	0.49	0.3194	-4.64	

q denotes the number of lags included in the modified ADF regression.

Table A2. Efficient Unit Root Test Results

$\ln(r_t \quad r_t^a)$: log user cost

Sample Period: 1967:1 – 2017:2

Divisia Aggregate	ADF-GLS	ADF-GLS

Table A3. Efficient Unit Root Test Results

$r_t \quad r_t^a$: user cost

Sample Period: 1967:1 – 2017:2

Divisia Aggregate	q	ADF-GLS Level	p-value	ADF-GLS Difference	p-value
M1	1	-2.17	0.0346	-9.32	0.0001
	2	-1.70	0.0968	-6.15	0.0001
	3	-1.97	0.0536	-4.85	0.0001
	4	-2.14	0.0388	-3.68	0.0002
M2	1	-2.41	0.0191	-8.72	0.0001
	2	-2.18	0.0359	-6.44	0.0001
	3	-2.25	0.0278	-4.88	0.0001
	4	-2.47	0.0145	-3.82	0.0001
MZM	1	-2.50	0.0164	-9.31	0.0001
	2	-2.11	0.0412	-6.48	0.0001
	3	-2.23	0.0287	-4.95	0.0001
	4	-2.41	0.0195	-3.81	0.0001
M4	1	-2.49	0.0159	-9.87	0.0001
	2	-2.24	0.0273	-7.67	0.0001
	3	-2.09	0.0414	-5.67	0.0001
	4	-2.23	0.0295	-4.26	0.0001

Notes: ADF-GLS is the modified Dickey-Fuller test statistic proposed by Elliot, Rothenberg, and Stock (1996). The p-values from testing the null hypothesis of a unit root in the level or first difference of the indicated series are bootstrapped following Benati (2015). q denotes the number of lags included in the modified ADF regression.

Table A4. Efficient Unit Root Test Results

v_t^a : consumption velocity

Sample Period: 1967:1 – 2017:2

Divisia Aggregate	q	ADF-GLS Level	p-value	ADF-GLS Difference	p-value
M1	1	-0.59	0.5248	-5.27	0.0001
	2	-0.67	0.4908	-4.60	0.0001
	3	-0.70	0.4749	-4.20	0.0001
	4	-0.70	0.4714	-3.92	0.0001
M2	1	-0.23	0.4682	-5.59	0.0001
	2	-0.29	0.4712	-4.84	0.0001
	3	-0.34	0.4667	-4.54	0.0001
	4	-0.32	0.4719	-3.98	

Table A 6. Efficient Unit Root Test Results

v_t^a : income velocity

Sample Period: 1967:1 – 2017:2

Table A8. Efficient Unit Root Test Results

$\ln(r_t \quad r_t^{a})$: log user cost

Sample Period: 1983:1 – 2017:2

Divisia Aggregate	q	ADF-GLS Level	p-value	ADF-GLS Difference	p-value
M1	1	-0.65	0.4098	-7.49	0.0001
	2	-0.73	0.3861	-5.65	0.0001
	3	-0.89	0.3301	-4.99	0.0001
	4	-0.90	0.3341	-4.11	0.0003
M2	1	-1.72	0.1063	-8.19	0.0001
	2	-1.76	0.0899	-6.39	0.0001
	3	-1.85	0.0731	-5.64	0.0001
	4	-1.82	0.0776	-4.66	0.0001
MZM	1	-1.86	0.0751	-8.36	0.0001
	2	-1.86	0.0725	-6.42	0.0001
	3	-1.93	0.0611	-5.54	0.0001
	4	-1.92	0.0611	-4.54	0.0001
M4	1	-2.71	0.0091	-10.18	0.0001
	2	-2.41	0.0184	-8.02	0.0001
	3	-2.30	0.0243	-6.77	0.0001
	4	-2.24	0.0314	-5.59	0.0001

Notes: ADF-GLS is the modified Dickey-Fuller test statistic proposed by Elliot, Rothenberg, and Stock (1996). The p-values from testing the null hypothesis of a unit root in the level or first difference of the indicated series are bootstrapped following Benati (2015). q denotes the number of lags included in the modified ADF regression.

Table A9. Efficient Unit Root Test Results

r_t r_t^a : user cost

Sample Period: 1983:1 – 2017:2

Divisia Aggregate	q	ADF-GLS Level	p-value	ADF-GLS Difference	p-value
M1	1	-0.83	0.3650	-6.59	0.0001
	2	-0.89	0.3429	-5.14	0.0001
	3	-0.98	0.3105	-4.31	0.0001
	4	-1.10	0.2574	-3.94	0.0001
M2	1	-1.94	0.0668	-7.57	0.0001
	2	-2.05	0.0473	-5.99	0.0001
	3	-2.18	0.0363	-4.97	0.0001
	4	-2.37	0.0224	-4.42	0.0001
MZM	1	-2.02	0.0540	-7.83	0.0001
	2	-2.05	0.0493	-6.08	0.0001
	3	-2.11	0.0410	-4.90	0.0001
	4	-2.29	0.0265	-4.19	0.0001
M4	1	-2.73	0.0073	-9.99	0.0001
	2	-2.47	0.0188	-8.13	0.0001
	3	-2.30	0.0261	-6.56	0.0001
	4	-2.35	0.0215	-5.36	0.0001

Notes: ADF-

Table A10. Efficient Unit Root Test Results

v_t^a

Table A11. Efficient Unit Root Test Results

$\ln(m_t^a)$: log money-output ratio

Sample Period: 1983:1 – 2017:2

Divisia Aggregate	q	ADF-GLS Level	p-value	ADF-GLS Difference	p-value
M1	1	1.00	0.7657	-4.27	0.0001
	2	0.91	0.7560	-3.57	0.0007
	3	0.64	0.7042	-3.18	0.0017
	4	0.52	0.6755	-3.11	0.0026
M2	1	-0.62	0.4923	-2.88	0.0058
	2	-0.63	0.4855	-2.67	0.0100
	3	-0.54	0.5239	-2.22	0.0322
	4	-0.61	0.5067	-2.00	0.0530
MZM	1	-0.89	0.3485	-4.29	0.0001
	2	-0.52	0.4578	-3.99	0.0002
	3	-0.46	0.4663	-3.64	0.0008
	4	-0.44	0.4674	-3.36	0.0011
M4	1	-1.63	0.1288	-3.89	0.0005
	2	-1.54	0.1514	-3.32	0.0009
	3	-1.55	0.1482	-3.25	0.0015
	4	-1.36	0.2024	-2.75	0.0071

Notes: The log money-output ratio is computed by dividing the indicated monetary aggregate by nominal GDP and taking the natural logarithm. ADF-GLS is the modified Dickey-Fuller test statistic proposed by Elliot, Rothenberg, and Stock (1996). The p-values from testing the null hypothesis of a unit root in the level or first difference of the indicated series are bootstrapped following Bena ti (2015). q denotes the number of lags included in the modified ADF regression.

Table A12. Efficient Unit Root Test Results

v_t^a : income velocity

Sample Period: 1983:1 – 2017:2

Divisia Aggregate	q	ADF-GLS Level	p-value	ADF-GLS Difference	p-value
M1	1	0.23	0.5902	-4.1314.6	

Table A13. Cointegrating Money Demand Relationships

Meltzer Specification: $\ln(m_t^a) = \alpha_0 - \alpha_1 \ln(r_t - r_t^a)$

$\ln(m_t^a)$: log money -output ratio

Sample Period: 1967:1 – 2017:2

Divisia Aggregate	q	α_0	α_1	Max Eigenvalue $r=0$	p-value	Max Eigenvalue $r=1$	p-value
M1	2*+	-6.69	1.94	44.81	0.0001	3.09	0.6588
	3	-7.06	1.98	39.05	0.0001	3.12	0.6658
	4	-6.62	1.78	26.80	0.0014	2.51	0.7545
	5	-6.33	1.66	21.04	0.0135	2.10	0.8330
M2	2*+	-4.93	1.55	44.20	0.0001	5.53	0.2561
	3	-4.98	1.57	43.44	0.0001	5.13	0.3019
	4	-5.00	1.58	33.50	0.0001	5.18	0.3038
	5	-5.12	1.67	30.70	0.0002	4.13	0.4378
MZM	2*+	-4.70	1.38	39.38	0.0001	5.95	0.2078
	3	-4.69	1.37	38.78	0.0001	5.43	0.2591
	4	-4.78	1.42	27.95	0.0007	5.57	0.2495
	5	-4.78	1.43	26.61	0.0012	4.49	0.3829
M4	2*+	-10.38	5.99	24.13	0.0026	8.62	0.0678
	3	-8.79	4.71	21.81	0.0055	8.54	0.0689
	4	-35.20	24.83	18.03	0.0309	8.07	0.0888
	5	-11.73	6.96	15.37	0.0792	6.81	0.1467

Notes: The log money -

Table A14. Cointegrating Money Demand Relationships

Cagan Specification: $\ln(m_t^a) = \delta_0 - \delta_1(r_t - r_t^a)$

$\ln(m_t^a)$: log money-output ratio

Sample Period: 1967:1 – 2017:2

Divisia Aggregate	Max Eigenvalue	Max Eigenvalue

Table A16 . Cointegrating Money Demand Relationships

Meltzer Specification: $\ln(m_t^a) = \alpha_0 - \alpha_1 \ln(r_t - r_t^a)$

$\ln(m_t^a)$: log money -output ratio -

Sample Period: 1983:1 – 2017:2

Divisia	Max Eigenvalue	Max Eigenvalue

Table A17 . Cointegrating Money Demand Relationships

Cagan Specification: $\ln(m$

Table A18. Cointegrating Money Demand Relationships

Selden-Latané Specification: $v_t^a = {}_0 + {}_1(r_t \quad r_t^a)$

v_t^a : income velocity

Sample Period: 1983:1 – 2017:2

Divisia	Max Eigenvalue	Max Eigenvalue
Aggregate Income a		